



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – STATISTICS

FIFTH SEMESTER – NOVEMBER 2023

UST 5501 – APPLIED STOCHASTIC PROCESSES

Date: 30-10-2023

Dept. No.

Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

Section A

Answer **ALL** the questions

10 x 2 = 20 marks

1. Define stochastic process and Markov chain.
2. Define transition probability matrix.
3. Write any two properties of periodicity.
4. Define recurrent and transient states.
5. State Abel lemma.
6. Write the postulates for a pure birth process.
7. State any two examples for renewal process.
8. Define excess life, current life in renewal process.
9. Write a note on extinction probability.
10. Explain branching process with the help of two generating functions.

Section B

Answer any **FIVE** questions

5 x 8 = 40 marks

- 11 Define classification of states and show that communication is an equivalence relation.
12. State and prove the necessary and sufficient condition for a state i to be recurrent.
13. Show that one-dimensional random walk is recurrent.
14. Consider the following one-step transition probabilities:
 $P_{00} = 1/6$, $P_{01} = 1/3$, $P_{02} = 1/2$, $P_{10} = P_{11} = 1/2$, $P_{20} = 1/6$,
 $P_{21} = 1/3$, $P_{22} = 1/2$, $P_{31} = 1/6$, $P_{32} = 1/3$, $P_{33} = 1/2$.
Find the stationary probability distribution.

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15. Derive $P_n(t)$ for Yule process assuming that $X(0) = N = 1$.
16. For birth and death process derive forward Kolmogorov differential equations.
17. Explain Type II counter model in renewal process.
18. Explain the following examples of Markov branching processes:
 - (a)Electron multipliers
 - (b)Neutron chain reaction
 - (c)Survival of family names. (3+3+2)

Section C

Answer any **TWO** questions

2 x 20 = 40 marks

19. Let a Markov chain on the states $\{0,1,2,3,4,5\}$ has the following one-step transition probabilities:
 $P_{00}=1/3, P_{01}=2/3, P_{10}=2/3, P_{11}=1/3, P_{22}=1/4, P_{23}=3/4, P_{32}=1/5, P_{33}=4/5,$
 $P_{40} = P_{42} = P_{44} = P_{45} = 1/4, P_{5j} = 1/6$ for $j = 0,1,2,3,4,5$.
 - (a) Find the equivalence classes. (2)
 - (b) Find the period for different classes. (2)
 - (c) Show that states 0,1,2,3 are recurrent and 4,5 are transient. (16)
20. Show that the three-dimensional random walk is transient.
21. (a) Explain age and block replacement policies. (10)
(b) State and prove the elementary renewal theorem. (10)
22. Establish the probability generating function relations for branching process and hence obtain mean and variance.

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